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DEPARTMENT OF STATISTICS
University of Wisconsin-Madison
1210 W. Dayton St.
Madison, WI 53706

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# MATHEMATICAL METHODS IN COMBATTING TERRORISM[1]

By

Bernard Harris

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Mathematical Methods in Combatting Terrorism

Bernard Harris

University of Wisconsin

Madison, Wisconsin, USA

**Abstract:** This document is a preliminary report on the role that mathematical and statistical methods might play in the defense against terrorist attacks. In no way does this replace the efforts of law enforcement agenices or intelligence activities. The hope is that mathematical techniques can make their efforts more efficient. The ideas enumerated here utilize the notion of Probabilistic Risk Analysis, which was developed for the purpose of assessing the safety of nuclear reactors, as well as randomization and game theory. More extensive work in these directions is contemplated for the future. The author is planning workshops to evaluate the ideas presented here and to elicit additional methodologies which may prove useful in this endeavor.

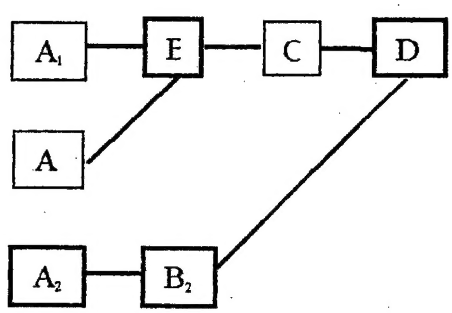
**1. Introduction and Summary.** The purpose of this report is to advance some thoughts of the author on the role that mathematical and statistical methods might play in combatting terrorist attacks. Clearly, this can not replace the efforts of law enforcement and intelligence agencies. However, mathematical and statistical methods may make such efforts more efficient. Possible benefits include methods for evaluating proposals for increased or modified security procedures. In addition, it may be possible to develop trchniques for allocating resources more efficiently. To this end, a panel has been appointed, which will meet in the near future to discuss such methods. The present membership of this panel is Professor Bernard Harris, the author of this report, Professor Vicki Bier, University of Wisconsin, Dr. Arthur Fries, Institue for Defense Analyses, Professor Torbjorn Thedeen, Center for Safety Research, Royal Technical University., Stockholm, Sweden, Professor Roger M.; Cooke, Technical University of Delft, Netherlands, and Dr. Lee Abramson, U. S. Nuclear Regulatory Commission. Some additional appointments may be made.

I will now outline how Probabilistic Risk Analysis (PRA) methods, game theory and randomization can play a role in combatting terrorism. It should be noted that these are not the only possible techniques from the mathematical sciences that may be employed. At the conclusion of this report, I will make some brief mention of a few other possibilities. I am less familiar with those and would want to delay serious consideration of them until a later time.

**2. The PRA Methodology.** The PRA method was initially developed for the purpose of assessing the safety of nuclear reactors. In some countries, this methodology is called PSA (Probabilistic Safety Analysis). The same methodology can be also adapted to study other

situations in which catastrophic failures are possible. Some illustrations include aircraft safety, maritme safety and highway safety.

As applied to nuclear reactors, a catalogue of accident sequences is produced and plotted in one of several possible ways ( fault trees and event trees are two of the most common). Probabilities are assigned to each of the nodes.These are then combined to obtain probabilistic estimates of the various accident sequences. This is a vaşt oversimplification, but retains the basic principles. There are several significant difficulties in implementing this methodology. In general, many of the events in the accident sequence are fairly rare events and their probabilities can not be estimated from data. Expert opinion is frequently employed as a method of eliciting probability estimates, but this is unreliable in the case of rare events . Bayesian methods are also frequently employed, but when there is very little data, the end result is very strongly influenced by the assumed prior distribution. There has been some success in assessing probabilities of events in accident sequences by identifying similar situations in other contexts and using to estimate failure probabilities. Also, the presence of stochastic dependence between failures is frequently difficult to assess accurately. Despite all of these difficulties in implementing the methodology, PRA methods have been fairly successful. The "bottom line" probability estimates should not be interpreted strictly, but in the sense of "order of magnitude" estimates. As such, the procedure has been successful in identifying the principal contributors to catastrophic failure and in identifying their relative importance. Below is a simplified event tree. The accident sequences ABCD, $A_1BCD$, $A_2B_2D$ result in catastrophic failure.




Thus, if the probabilities of failures and the dependencies of the various components are known (which is rarely the case), it is possible to determine the probability of failure for a specified accident sequence.

To summarize, a PRA study is basically a catalogue of possible accident sequences and estimates of the probabilities of failure for each of them. PRA studies are now fairly universally employed in the study of catastrophic failures. However, the history of this methodology is filled with controversies.

Specifically, during the 1970's, the government commissioned Professor Norman Rasmussen of the Massachussets Institute of Technology to devise a method for evaluating the safety of nuclear reactors. His study [1] (officially known as WASH-1400) was the prototype for PRA studies. However, many of the statistical and probabilistic methods employed in this study were flawed or controversial. This resulted in the government

establishing a review panel known as the "Lewis Commission", which issued a report critical of the Rasmussen report. Indcidentally, the "Lewis Commission" report [2] was also flawed. However, the PRA method [3] went through many revisions and is now a basic method in evaluating risks of catastrophic failures.

In the present context (terrorist attacks), the above event diagram can be regarded as a security system. If a security failure occurs at any node, there is a security breach. However, the terrorist may be prevented from carrying out his attack if the security procedure succeeds at the next node. If all nodes in a path fail, then the terrorist attack is successful. Thus the following paths represent successful terrorist attacks, ABCD, $A_1BCD$, $A_2B_2D$. If the (conditional) probabilities of detecting the terrorist at each node can be evaluated, then the probability that this security method will fail can be determined. In this manner, the PRA methology can be utilized to study the efficacy of security procedures. The above analysis is, of course, a gross oversimplification of the problem and a careful analysis will require a substantial expenditure of time and resources.

Thus, to summarize the use of PRA methods, the proposal is to combine systems analysis techniques to describe security procedures with statistical methods used in systems reliability. Then, after determining the probability estimates for each node and the stochastic dependencies between nodes, the effectiveness of the security procedure can be determined. Inasmuch as probability estimates for some of the nodes may not be ascertainable, it may be possible to obtain estimates using precursor methods to extend estimates for partial failures to complete failures. There is an extensive literature on precursor methods, which will not be described here.

**3. The Magnitude of the Problem.** If one lists the various targets that terrorists may attack, the enormous magnitude of the problem becomes evident. A partial list of vulnerabilities is described below.

Transportation and shipping.
- Air transport
  - Passenger
  - Freight
- Rail transport
  - Passenger
  - Freight
- Maritime
- Road
  - Highway
  - Bridges
- Local
  - Bridges
  - Ferries
  - Subways
  - Buses

Buildings
- Industrial
- Retail
- Schools
- Residential
- Government

Agriculture
Water Supply
Communications

One of the scenarios that I find particularly troublesome is what I would call multiple attacks. Such can be particularly unnerving to a population. An example of this might be the following: Bomb a film theater in Des Moines on Monday, a shopping mall in Omaha on Tuesday, a department store in Wichita on Wednesday, and so on.

In addition, there are also the possibilities of biological attacks, chemical attacks and cyber attacks.

What role can mathematical methods play in such a situation? An initial thought is that game theoretic methods can be appropriate. One can expect that a terrorist organization will avoid well defended targets and attack less well defended targets. As noted in the next paragraph, randomization is needed, so that the terrorist will not know in advance which potential targets are less well defended. Unfortunately, the game theorical models needed to describe such problems are not zero-sum two person games, which is by far, the simplest case to analyze. Some study is needed to assess the relative utilities employed by the terrorists and the defenders. It is also expected that such analyses will be controversial.

Randomization should also play a well defined role. Given the list of potential targets described above, it will be difficult, if not impossible for law enforcement and security personnel to protect all of these. One way to make it more difficult for terrorist organizations to succeed is to use randomization in assigning security personnel to potential targets, so that which targets will be vulnerable can not be predicted by a terrorist organization.

There are many illustrations of the failure by law enforcement to understand the principle of randomization. I would like to cite two illustrations that I have noted. These illustrations have been somewhat simplified, but the basic priciple is maintained.

In one district of a city, there is a two hour limit for parking motor vehicles. The parking enforcement personnel arrive each day at 1100 and begin to mark parked vehicles, returning two hours later to issue citations. Thus if you wish to park illegally, arrive after 1300 and you will not be penalized.

The security personnel sweep through a building containing expensive electronic communications equipment each morning at 0230, leaving about one hour later. If you should wish to steal such equipment, you will avoid detection by arriving after 0330.

**4. LPHC Problems.** Many of the problems alluded to in the above material belong to an area of statisical inference known as LPHC (low probability high consequence) problems. A large fraction of the standard statistical techniques is inapplicable for such problems. To illustrate this, consider some of the customary loss functions used in point estimation. These are

$$L(\theta,\hat{\theta}) = (\theta - \hat{\theta})^2,$$

or

$$L(\theta,\hat{\theta}) = |\theta - \hat{\theta}|,$$

where $\hat{\theta}$ is a point estimator of $\theta$ (possibly vector valued) , a parameter of the probability distribution from which the sample data is obtained. To see that such loss functions are inappropriate, consider the following context which is applicable to the present situation. Let $\theta$ denote the reliability in a situation in which failures may be catastrophic. Let $\hat{\theta}$ be a point estimator the reliability, $0 \le \theta,\hat{\theta} \le 1$. Should $\theta$=.5, then you are in a disastrous situation and very larger errors of estimation have little effect on decision making. On the other hand, if the probability of a catastrophic failure is .01 and your estimate is .001, then you get a serious problem one time in a hundred and you expect such a problem one time in as thousand. Consequently, your "cost" is really ten times greater than anticipated. Thus the behavior in the neighborhood of unity should be critical in decision making. This suggests loss functions of the form

$$L(\theta,\hat{\theta}) = |(1-\theta)^{-1}-(1-\hat{\theta})^{-1}|$$

or

$$\frac{|\theta - \hat{\theta}|^{\nu}}{(1-\theta)^{\kappa}}, \; \nu, \kappa > 0.$$

The effect of such loss functions is that squared error (or absolute error) does not play its usual role and central limit theorem and the normal distribution are not significant. Outliers play a major role, since extreme observations may be the most informative. That is, if data is around the average, then there is no significant evidence of catastrophic failure. On the other hand, extreme values can call attention to dangerous situations.

**5. Other possible methodologies.** The following possibilities should also be considered.
1.Data mining and cluster analysis, however, an expected by-product of such methods may be racial profiling in investigation of potential terrorist suspects.
2. Large scale simulations for the purpose of studying potential security initiatives.
3. Operations research methods of the type utilized during the second world war.

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